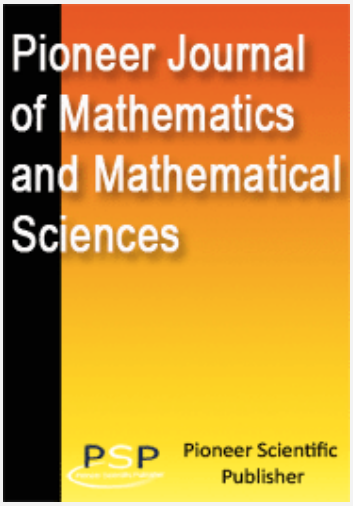
Pioneer Journal of Mathematics and Mathematical Sciences
Volume 13, Issue 1, Pages 15-37 (January 2015)

ISSN: 2230-9829



A NOTE ON $f^{\pm}$-ZAGREB INDICES IN RESPECT OF JACO GRAPHS, $J_n(1)$, $n \in \mathbb{N}$ AND THE INTRODUCTION OF KHAZAMULA IRREGULARITY

Johan Kok and Vivian Mukungunugwa

Received October 1, 2014

Abstract

The topological graph indices $irr(G)$ related to the first Zagreb index, $M_1(G)$ and the second Zagreb index, $M_2(G)$ are of the oldest irregularity measures researched. Alberton [M. O. Albertson, The irregularity of a graph, Ars Combinatoria 46 (1997), 219-225] introduced the irregularity of G as

$$irr(G) = \sum_{e \in E(G)} imb(e), \quad imb(e) = |\, d(v) - d(u) \,|_{e=vu}.$$

In the paper of Fath-Tabar [G. H. Fath-Tabar, Old and new Zagreb indices of graphs, MATCH Communications in Mathematical and in Computer Chemistry 65 (2011), 79-84], Alberton's indice was named the third Zagreb indice to conform with the terminology of chemical graph theory. Recently Ado et al. [H. Abdo and D. Dimitrov, The total irregularity of a graph, arXiv: 1207.5267v1 [math.CO], 24 July 2012] introduced the topological indice called total irregularity. The latter could be called the fourth Zagreb indice. We define the $\pm$Fibonacci weight, $f_i^{\pm}$ of a vertex v_i to be $-f_{d(v_i)}$, if $d(v_i)$ is uneven and, $f_{d(v_i)}$, if $d(v_i)$ is even. From the aforesaid we define the $f^{\pm}$-Zagreb indices. This paper presents introductory results for the undirected underlying graphs of Jaco graphs, $J_n(1)$, $n \leq 12$. For more on Jaco graphs $J_n(1)$ see [J. Kok, P. Fisher, B. Wilkens, M. Mabula and V. Mukungunugwa, Characteristics of Finite Jaco Graphs, $J_n(1)$, $n \in \mathbb{N}$, arXiv: 1404.0484v1 [math.CO], 2 April 2014; J. Kok, P. Fisher, B. Wilkens, M. Mabula and V. Mukungunugwa, Characteristics of Jaco Graphs, $J_\infty(a)$, $a \in \mathbb{N}$, arXiv: 1404.1714v1 [math.CO], 7 April 2014]. Finally, we introduce the Khazamula irregularity as a new topological variant. We also present five open problems.

Keywords and phrases: total irregularity, irregularity, imbalance, Zagreb indices, $\pm$Fibonacci weight, total f-irregularity, Fibonaccian irregularity, $f^{\pm}$-Zagreb indices, Jaco graphs, Zeckendorf representation, Khazamula irregularity, Khazamula theorem.